Chapter 9

1

Linear Momentum and Collisions

1

Lecture 03

2

Collisions

2

Collisions – Characteristics

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The term collision represents an event during which two particles come close to each other and interact by means of forces.

- May involve physical contact, but must be generalized to include cases with interaction without physical contact

The interaction forces are assumed to be much greater than any external forces present.

- This means the impulse approximation can be used.

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Collisions – Example 1

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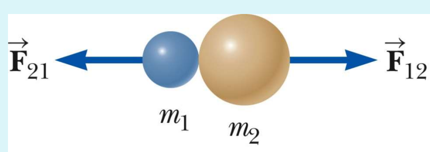
- Collisions may be the result of direct contact.
- The impulsive forces may vary in time in complicated ways.
- Momentum is conserved.



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Collisions – Example 2

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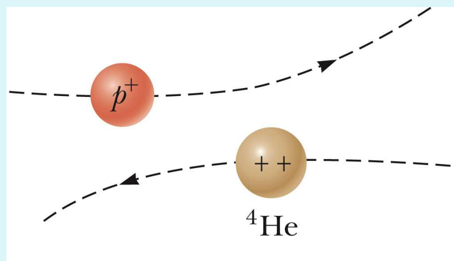
- The collision need not include physical contact between the objects.
- There are still forces between the particles.
- This type of collision can be analyzed in the same way as those that include physical contact.



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Types of Collisions

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- **Elastic collision**
  - Both momentum and kinetic energy are conserved
- **Inelastic collision**
  - kinetic energy is not conserved
  - momentum is still conserved.
- **Perfectly inelastic collision**
  - kinetic energy is not conserved
  - momentum is still conserved.
  - the objects stick together after the collision

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Types of Collisions

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Elastic collisions occur on a microscopic level.

- In macroscopic collisions, only approximately elastic collisions actually occur.
- Generally some energy is lost to deformation, sound, etc.

In an inelastic collision, some kinetic energy is lost, but the objects do not stick together.

Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types .

Momentum is conserved in all collisions

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Perfectly Inelastic Collisions

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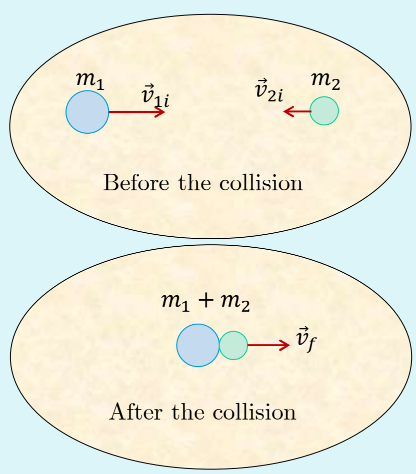
- Momentum of an isolated system is conserved in any collision, so the total momentum before the collision is equal to the total momentum of the composite system after the collision.
- Since the objects stick together, they share the same velocity after the collision.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1 + m_2)\vec{v}_f$$



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Elastic Collisions

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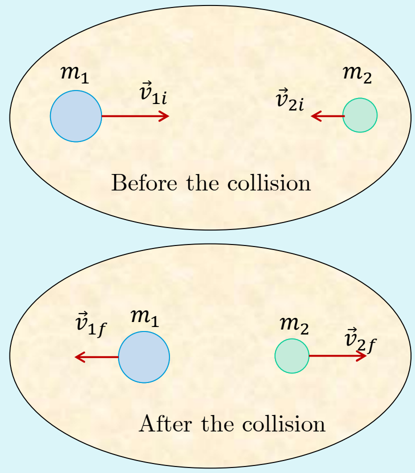
- Both momentum and kinetic energy are conserved.

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

- Typically, there are two unknowns to solve for and so you need two equations.



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Elastic Collisions

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Equations of conservation of linear momentum and conservation of kinetic energy can be solved for the final velocities in terms of the initial velocities and masses:

$$\vec{v}_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{2i}$$

$$\vec{v}_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{2i}$$

Can only be used with a one-dimensional, elastic collision.

Remember to use the appropriate signs for all velocities.

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Elastic Collisions, Some special cases

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$m_1 = m_2$; the particles exchange velocities.

- $v_{1f} = v_{2i}$
- $v_{2f} = v_{1i}$

When a very heavy particle (m_1) collides head-on with a very light one (m_2) initially at rest $(m_1 \gg m_2$ and $v_{2i} = 0)$, the heavy particle continues in motion unaltered and the light particle rebounds with a speed of about twice the initial speed of the heavy particle.

- $v_{1f} \cong v_{1i}$
- $v_{2f} \cong 2v_{1i}$

When a very light particle (m_1) collides head-on with a very heavy particle (m_2) initially at rest $(m_1 \ll m_2$ and $v_{2i} = 0)$, the light particle has its velocity reversed and the heavy particle remains approximately at rest.

- $v_{1f} = -v_{1i}$
- $v_{2f} \cong 0$

Two-Dimensional Collisions

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- Same as in 1-D.
- Vector Momentum Conservation:

$$m_1\vec{v}_{1x} + m_2\vec{v}_{2xi} = m_1\vec{v}_{1x} + m_2\vec{v}_{2xf}$$

$$m_1\vec{v}_{1yi} + m_2\vec{v}_{2yi} = m_1\vec{v}_{1yf} + m_2\vec{v}_{2yf}$$

- If the collision is elastic, Kinetic Energy Conservation:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

  - The simpler equations can only be used for one-dimensional situations.

Two-Dimensional Collision, example

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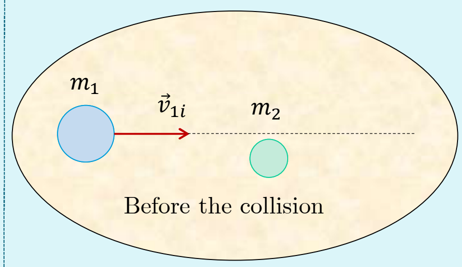
Particle 1 is moving at velocity $\vec{v}_{1i}$ and particle 2 is at rest.

Before the collision:

- The initial momentum in the *x*-direction, is:

$$m_1\vec{v}_{1i}$$

- The initial momentum in the *y*-direction, is 0.



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Two-Dimensional Collision, example cont.

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After the collision:

- The momentum in the *x*-direction is:

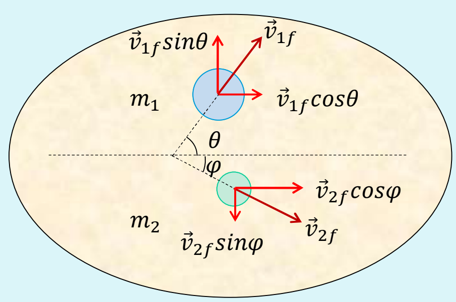
$$m_1\vec{v}_{1f}cos\theta + m_2\vec{v}_{2f}cos\varphi$$

- After the collision, the momentum in the *y*-direction is

$$m_1\vec{v}_{1f}sin\theta - m_2\vec{v}_{2f}sin\varphi$$

  - The negative sign is due to the component of the velocity being downward.

- If the collision is elastic, apply the kinetic energy equation.



After the collision

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1-D elastic collision

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Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A block of mass $m_1 = 1.6\ kg$ initially moving to the right with a speed of $4\ m/s$ on a frictionless, horizontal track collides with a light spring attached to a second block of mass $m_2 = 2.1\ kg$ initially moving to the left with a speed of $2.5\ m/s$. The spring constant is $600\ N/m$.

- Find the velocities of the two blocks after the collision.
- Determine the velocity of block 2 during the collision, at the instant block 1 is moving to the right with a velocity of $3\ m/s$.
- Determine the distance the spring is compressed at that instant.

(NOTE: Because the spring force is conservative, kinetic energy in the system of two blocks and the spring is not transformed to internal energy during the compression of the spring. We can categorize the collision as being elastic.)

Analyze Because momentum of the system is conserved, apply Equation 9.16:

$$(1)\quad m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Because the collision is elastic, apply Equation 9.20:

$$(2)\quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

Multiply Equation (2) by m_1:

$$(3)\quad m_1v_{1i} - m_1v_{2i} = -m_1v_{1f} + m_1v_{2f}$$

Add Equations (1) and (3):

$$2m_1v_{1i} + (m_2 - m_1)v_{2i} = (m_1 + m_2)v_{2f}$$

Solve for v_{2f}:

$$v_{2f} = \frac{2m_1v_{1i} + (m_2 - m_1)v_{2i}}{m_1 + m_2}$$

Substitute numerical values:

$$v_{2f} = \frac{2(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg} - 1.60\text{ kg})(-2.50\text{ m/s})}{1.60\text{ kg} + 2.10\text{ kg}} = \boxed{3.12\text{ m/s}}$$

Solve Equation (2) for v_{1f} and substitute numerical values:

$$v_{1f} = v_{2f} - v_{1i} + v_{2i} = 3.12\text{ m/s} - 4.00\text{ m/s} + (-2.50\text{ m/s}) = \boxed{-3.38\text{ m/s}}$$

Analyze Apply Equation 9.16:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Solve for v_{2f}:

$$v_{2f} = \frac{m_1v_{1i} + m_2v_{2i} - m_1v_{1f}}{m_2}$$

Substitute numerical values:

$$v_{2f} = \frac{(1.60\text{ kg})(4.00\text{ m/s}) + (2.10\text{ kg})(-2.50\text{ m/s}) - (1.60\text{ kg})(3.00\text{ m/s})}{2.10\text{ kg}} = \boxed{-1.74\text{ m/s}}$$

Write the appropriate reduction of Equation 8.2:

$$\Delta K + \Delta U = 0$$

Evaluate the energies, recognizing that two objects in the system have kinetic energy and that the potential energy is elastic:

$$\left[\left(\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2\right) - \left(\tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2\right)\right] + \left(\tfrac{1}{2}kx^2 - 0\right) = 0$$

Solve for x^2:

$$x^2 = \tfrac{1}{k}\left[m_1(v_{1i}^2 - v_{1f}^2) + m_2(v_{2i}^2 - v_{2f}^2)\right]$$

Substitute numerical values:

$$x^2 = \left(\frac{1}{600\text{ N/m}}\right)\{(1.60\text{ kg})[(4.00\text{ m/s})^2 - (3.00\text{ m/s})^2] + (2.10\text{ kg})[(2.50\text{ m/s})^2 - (1.74\text{ m/s})^2]\}$$

$$\rightarrow x = \boxed{0.173\text{ m}}$$

1-D perfectly inelastic collision

Saturday, 30 January, 2021 15:28

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
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An $1800\ kg$ car stopped at a traffic light is struck from the rear by a $900\ kg$ car. The two cars become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at $20\ m/s$ before the collision, what is the velocity of the entangled cars after the collision?

Use the isolated system model for momentum: $\Delta\vec{\mathbf{p}} = 0 \quad \rightarrow \quad p_i = p_f \quad \rightarrow \quad m_1 v_i = (m_1 + m_2) v_f$

Solve for v_f and substitute numerical values: $v_f = \dfrac{m_1 v_i}{m_1 + m_2} = \dfrac{(900\text{ kg})(20.0\text{ m/s})}{900\text{ kg} + 1\,800\text{ kg}} = \boxed{6.67\text{ m/s}}$

1-D inelastic collision

Saturday, 30 January, 2021 15:30

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A $1200\ kg$ car traveling initially at $v_{Ci} = 25\ m/s$ in an easterly direction crashes into the back of a $9000\ kg$ truck moving in the same direction at $v_{Ti} = 20\ m/s$. The velocity of the car immediately after the collision is $v_{Cf} = 18\ m/s$ to the east.

- What is the velocity of the truck immediately after the collision?
- What is the change in mechanical energy of the car–truck system in the collision?
- Account for this change in mechanical energy.

P9.22 (a) Conservation of momentum gives

$$m_T v_{Tf} + m_C v_{Cf} = m_T v_{Ti} + m_C v_{Ci}$$

Solving for the final velocity of the truck gives

$$v_{Tf} = \frac{m_T v_{Ti} + m_C\left(v_{Ci} - v_{Cf}\right)}{m_T}$$

$$= \frac{(9\ 000\ \text{kg})(20.0\ \text{m/s}) + (1\ 200\ \text{kg})[(25.0 - 18.0)\ \text{m/s}]}{9\ 000\ \text{kg}}$$

$$v_{Tf} = \boxed{20.9\ \text{m/s East}}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta KE = KE_f - KE_i = \left[\frac{1}{2}m_C v_{Cf}^2 + \frac{1}{2}m_T v_{Tf}^2\right] - \left[\frac{1}{2}m_C v_{Ci}^2 + \frac{1}{2}m_T v_{Ti}^2\right]$$

$$= \frac{1}{2}\left[m_C\left(v_{Cf}^2 - v_{Ci}^2\right) + m_T\left(v_{Tf}^2 - v_{Ti}^2\right)\right]$$

$$= \frac{1}{2}\left\{(1\ 200\ \text{kg})\left[(18.0\ \text{m/s})^2 - (25.0\ \text{m/s})^2\right] + (9\ 000\ \text{kg})\left[(20.9\ \text{m/s})^2 - (20.0\ \text{m/s})^2\right]\right\}$$

$$\Delta KE = \boxed{-8.68 \times 10^3\ \text{J}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

(c) The mechanical energy of the car-truck system has decreased. Most of the energy was transformed to internal energy with some being carried away by sound.

The Ballistic Pendulum

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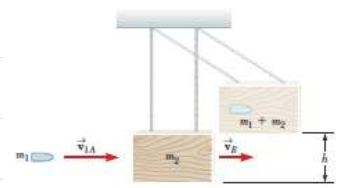
A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we determine the speed of the projectile from a measurement of h?



Just before and after the collision; the linear momentum is conserved:

$\Rightarrow\ mv_{1i} + Mv_{2i} = (m+M)v_f\ ;\ \{v_{2i} = 0\ m/s\}$

$\Rightarrow\ v_f = \dfrac{mv_{1i}}{(m+M)}$ .....(1)

After the collision; the mechanical energy is conserved:

$E_f = E_i\ \Rightarrow\ U_f = K_i$

$\Rightarrow\ (m+M)gh = \dfrac{1}{2}(m+M)v_i^2\ \Rightarrow\ v_i = \sqrt{2gh}$ .....(2)

$\{v_i = v_f$ just before the collision$\}$

$\Rightarrow\ v_{1i} = \left(\dfrac{m+M}{m}\right)\sqrt{2gh}$

Noting that $v_{2A} = 0$, solve Equation 9.15 for v_B: (1) $v_B = \dfrac{m_1 v_{1A}}{m_1 + m_2}$

Categorize For the process during which the projectile–block combination swings upward to height h (ending at a configuration we'll call C), we focus on a *different* system, that of the projectile, the block, and the Earth. We categorize this part of the problem as one involving an *isolated system* for *energy* with no nonconservative forces acting.

Analyze Write an expression for the total kinetic energy of the system immediately after the collision: (2) $K_B = \frac{1}{2}(m_1 + m_2)v_B^{\,2}$

Substitute the value of v_B from Equation (1) into Equation (2): $K_B = \dfrac{m_1^{\,2} v_{1A}^{\,2}}{2(m_1 + m_2)}$

This kinetic energy of the system immediately after the collision is *less* than the initial kinetic energy of the projectile as is expected in an inelastic collision.

We define the gravitational potential energy of the system for configuration B to be zero. Therefore, $U_B = 0$, whereas $U_C = (m_1 + m_2)gh$.

Apply the isolated system model to the system: $\Delta K + \Delta U = 0\ \rightarrow\ (K_C - K_B) + (U_C - U_B) = 0$

Substitute the energies:

$$\left(0 - \frac{m_1^{\,2} v_{1A}^{\,2}}{2(m_1 + m_2)}\right) + [(m_1 + m_2)gh - 0] = 0$$

Solve for v_{1A}: $v_{1A} = \left(\dfrac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$

2-D elastic collision

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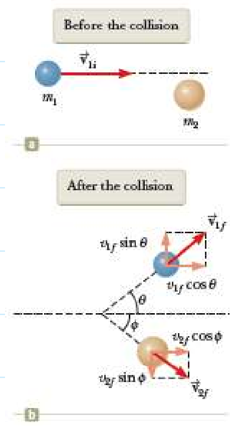
A proton collides elastically with another proton that is initially at rest. The incoming proton has an initial peed of $3.5\times10^5\ m/s$ and makes a glancing collision with the second proton as shown. After the collision, one proton moves off at an angle of 37° to the original direction of motion and the second deflects at an angle of φ to the same axis. Find the final speeds of the two protons and the angle φ.



Analyze Using the isolated system model for both momentum and energy for a two-dimensional elastic collision, set up the mathematical representation with Equations 9.25 through 9.27:

(1) $v_{1i} = v_{1f}\cos\theta + v_{2f}\cos\phi$

(2) $0 = v_{1f}\sin\theta - v_{2f}\sin\phi$

(3) $v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

Rearrange Equations (1) and (2):

$$v_{2f}\cos\phi = v_{1i} - v_{1f}\cos\theta$$
$$v_{2f}\sin\phi = v_{1f}\sin\theta$$

Square these two equations and add them:

$$v_{2f}^2\cos^2\phi + v_{2f}^2\sin^2\phi = v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta + v_{1f}^2\cos^2\theta + v_{1f}^2\sin^2\theta$$

Incorporate that the sum of the squares of sine and cosine for *any* angle is equal to 1:

(4) $v_{2f}^2 = v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta + v_{1f}^2$

Substitute Equation (4) into Equation (3):

$$v_{1f}^2 + (v_{1i}^2 - 2v_{1i}v_{1f}\cos\theta + v_{1f}^2) = v_{1i}^2$$

(5) $v_{1f}^2 - v_{1i}v_{1f}\cos\theta = 0$

One possible solution of Equation (5) is $v_{1f} = 0$, which corresponds to a head-on, one-dimensional collision in which the first proton stops and the second continues with the same speed in the same direction. That is not the solution we want.

Divide both sides of Equation (5) by v_{1f} and solve for the remaining factor of v_{1f}:

$$v_{1f} = v_{1i}\cos\theta = (3.50\times10^5\ \text{m/s})\cos 37.0^\circ = \boxed{2.80\times10^5\ \text{m/s}}$$

Use Equation (3) to find v_{2f}:

$$v_{2f} = \sqrt{v_{1i}^2 - v_{1f}^2} = \sqrt{(3.50\times10^5\ \text{m/s})^2 - (2.80\times10^5\ \text{m/s})^2} = \boxed{2.11\times10^5\ \text{m/s}}$$

Use Equation (2) to find ϕ:

(2) $$\phi = \sin^{-1}\left(\frac{v_{1f}\sin\theta}{v_{2f}}\right) = \sin^{-1}\left[\frac{(2.80\times10^5\ \text{m/s})\sin 37.0^\circ}{(2.11\times10^5\ \text{m/s})}\right] = \boxed{53.0^\circ}$$

Finalize It is interesting that $\theta + \phi = 90^\circ$. This result is *not* accidental. Whenever two objects of equal mass collide elastically in a glancing collision and one of them is initially at rest, their final velocities are perpendicular to each other.

2-D perfectly inelastic collision

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- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
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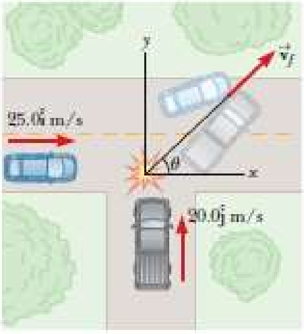
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

A $1500\ kg$ car traveling east with a speed of $25\ m/s$ collides at an intersection with a $2500\ kg$ truck traveling north at a speed of $20\ m/s$, as shown. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.



Apply the isolated system model for momentum in the x direction:

$$\Delta p_x = 0 \rightarrow \sum p_{xi} = \sum p_{xf} \rightarrow (1) \quad m_1 v_{1i} = (m_1 + m_2) v_f \cos\theta$$

Apply the isolated system model for momentum in the y direction:

$$\Delta p_y = 0 \rightarrow \sum p_{yi} = \sum p_{yf} \rightarrow (2) \quad m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta$$

Divide Equation (2) by Equation (1):

$$\frac{m_2 v_{2i}}{m_1 v_{1i}} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Solve for θ and substitute numerical values:

$$\theta = \tan^{-1}\left(\frac{m_2 v_{2i}}{m_1 v_{1i}}\right) = \tan^{-1}\left[\frac{(2\,500\text{ kg})(20.0\text{ m/s})}{(1\,500\text{ kg})(25.0\text{ m/s})}\right] = \boxed{53.1^\circ}$$

Use Equation (2) to find the value of v_f and substitute numerical values:

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2)\sin\theta} = \frac{(2\,500\text{ kg})(20.0\text{ m/s})}{(1\,500\text{ kg} + 2\,500\text{ kg})\sin 53.1^\circ} = \boxed{15.6\text{ m/s}}$$

2-D inelastic collision

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- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
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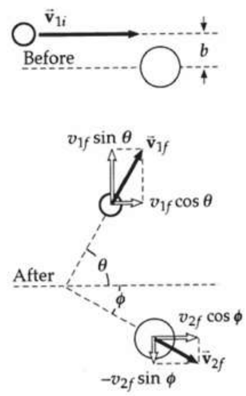
A $0.3\ kg$ puck, initially at rest on a horizontal, frictionless surface, is struck by a $0.2\ kg$ puck moving initially along the x axis with a speed of $2\ m/s$. After the collision, the $0.2\ kg$ puck has a speed of $1\ m/s$ at an angle of $\theta = 53^{\circ}$ to the positive x axis.

- Determine the velocity of the $0.3\ kg$ puck after the collision.
- Find the fraction of kinetic energy transferred away or transformed to other forms of energy in the collision.

***P9.35** (a) We write equations expressing conservation of the x and y components of momentum, with reference to the figures on the right. Let the puck initially at rest be m_2. In the x direction,



ANS. FIG. P9.35

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

which gives

$$v_{2f}\cos\phi = \frac{m_1 v_{1i} - m_1 v_{1f}\cos\theta}{m_2}$$

or

$$v_{2f}\cos\phi = \left(\frac{1}{0.300\text{ kg}}\right)[(0.200\text{ kg})(2.00\text{ m/s}) - (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0°]$$

In the y direction,

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

which gives

$$v_{2f}\sin\phi = \frac{m_1 v_{1f}\sin\theta}{m_2}$$

or

$$0 = (0.200\text{ kg})(1.00\text{ m/s})\sin 53.0° - (0.300\text{ kg})(v_{2f}\sin\phi)$$

From these equations, we find

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.532}{0.932} = 0.571 \quad \text{or} \quad \boxed{\phi = 29.7°}$$

Then $v_{2f} = \dfrac{0.160\text{ kg}\cdot\text{m/s}}{(0.300\text{ kg})(\sin 29.7°)} = \boxed{1.07\text{ m/s}}$

(b) $K_i = \dfrac{1}{2}(0.200\text{ kg})(2.00\text{ m/s})^2 = 0.400\text{ J}$ and

$$K_f = \frac{1}{2}(0.200\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(0.300\text{ kg})(1.07\text{ m/s})^2 = 0.273\text{ J}$$

$$f_{\text{lost}} = \frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i} = \frac{0.273\text{ J} - 0.400\text{ J}}{0.400\text{ J}} = \boxed{-0.318}$$